$C_d(n)$ = The Number of Visually Distinguishable Reachable Configurations of an $n^d$ Rubik's Cube

$$C_4(n)=\frac{15!\cdot 12^{15}}{6}\left(24!\cdot 32!\cdot 2^{26}\cdot 6^{33}\right)^{(n \bmod 2)}\left(\frac{64!}{2}\cdot 3^{63}\right)^{\left\lfloor \frac{n-2}{2}\right\rfloor}\left(\frac{96!}{24^{24}}\cdot 2^{95}\right)^{\left\lfloor \frac{n-2}{2}\right\rfloor+(n \bmod 2)\left(\frac{n-3}{2}\right)}\cdot$$

$$\left(\frac{192!}{24^{48}}\right)^{\frac{\left\lfloor \frac{n-4}{2}\right\rfloor\left\lfloor \frac{n-2}{2}\right\rfloor}{2}}\left(\frac{64!}{(8!)^{8}}\right)^{\left\lfloor \frac{n-2}{2}\right\rfloor}\left(\frac{96!}{(12!)^{8}}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\left(\frac{48!}{(6!)^{8}}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\cdot$$

$$\left(\frac{192!}{(24!)^{8}}\right)^{\frac{\left\lfloor \frac{n-4}{2}\right\rfloor\left\lfloor \frac{n-2}{2}\right\rfloor}{2}+\frac{(n \bmod 2)(n-5)(n-3)(n-1)+\left|(n \bmod 2)-1\right|(n-4)(n-3)(n-2)}{24}}$$

$$C_5(n)=\frac{31!}{2}\cdot 60^{31}\left[(80!)^2(40!)\cdot 60\cdot 24^{80}\cdot 6^{80}\cdot 2^{40}\right]^{n \bmod 2}\left(\frac{160!}{6}\cdot 12^{160}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor}\cdot$$

$$\left(\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor+(n \bmod 2)\binom{n-3}{2}}\left(\frac{640!}{24^{160}}\cdot 3^{639}\right)^{\frac{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{2}}\left(\frac{320!}{(8!)^{40}}\cdot 2^{319}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor}\cdot$$

$$\left(\frac{480!}{(12!)^{40}}\cdot 2^{479}\right)^{(n \bmod 2)\binom{n-3}{2}}\left(\frac{960!}{(24!)^{40}}\cdot 2^{959}\right)^{\frac{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{2}+\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor\cdot 2^{(n \bmod 2)-1}}\cdot$$

$$\left(\frac{240!}{(6!)^{40}}\cdot 2^{239}\right)^{(n \bmod 2)\binom{n-3}{2}}\left(\frac{1920!}{(24!)^{80}}\right)^{\frac{\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{6}}\left(\frac{160!}{(16!)^{10}}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor}\cdot$$

$$\left(\frac{320!}{(32!)^{10}}\right)^{(n \bmod 2)\binom{n-3}{2}}\left(\frac{640!}{(64!)^{10}}\right)^{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}\left(\frac{240!}{(24!)^{10}}\right)^{(n \bmod 2)\binom{n-3}{2}}\cdot$$

$$\left(\frac{960!}{(96!)^{10}}\right)^{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor\cdot 2^{(n \bmod 2)-1}+(n \bmod 2)\left\lfloor\frac{(n-5)(n-3)}{8}\right\rfloor}\left(\frac{80!}{(8!)^{10}}\right)^{(n \bmod 2)\binom{n-3}{2}}\cdot$$

$$\left(\frac{1920!}{(192!)^{10}}\right)^{\frac{\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{6}\left(2^{n \bmod 2}+2\right)+\frac{\left\lfloor\frac{n-8}{2}\right\rfloor\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{12}}\left(\frac{480!}{(48!)^{10}}\right)^{(n \bmod 2)\left\lfloor\frac{(n-5)(n-3)}{8}\right\rfloor}$$

$$C_6(n)=\frac{63!}{2}\cdot 360^{63}\left(360\cdot 192!\cdot 120^{192}\cdot 240!\cdot 24^{240}\cdot 160!\cdot 60!\cdot 6^{160}\cdot 2^{59}\right)^{n \bmod 2}\left(\frac{384!}{2}\cdot 60^{384}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor}\cdot$$

$$\left(\frac{960!}{2}\cdot 24^{720}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor+(n \bmod 2)\left(\frac{n-3}{2}\right)}\left(\frac{1920!}{24^{480}}\cdot\frac{12^{1920}}{3}\right)^{\frac{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{2}}\left(\frac{1280!}{(8!)^{160}}\cdot\frac{6^{1280}}{2}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor}\left(\frac{1920!}{(12!)^{160}}\cdot\frac{6^{1920}}{2}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\cdot$$

$$\left(\frac{3840!}{(24!)^{160}}\cdot\frac{6^{3840}}{2}\right)^{\frac{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{2}+\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor\cdot 2^{(n \bmod 2)-1}}\left(\frac{960!}{(6!)^{160}}\cdot\frac{6^{960}}{2}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\left(\frac{7680!}{(24!)^{320}}\cdot 3^{7679}\right)^{\frac{\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{6}}\cdot$$

$$\left(\frac{960!}{(16!)^{60}}\cdot 2^{959}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor}\left(\frac{1920!}{(32!)^{60}}2^{1919}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\left(\frac{3840!}{(64!)^{60}}\cdot 2^{3839}\right)^{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}\left(\frac{1440!}{(24!)^{60}}\cdot 2^{1439}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\cdot$$

$$\left(\frac{5760!}{(96!)^{60}}\cdot 2^{5759}\right)^{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor\cdot 2^{(n \bmod 2)-1}+(n \bmod 2)\left[\frac{(n-5)(n-3)}{8}\right]}\left(\frac{480!}{(8!)^{60}}\cdot 2^{479}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\cdot$$

$$\left(\frac{11520!}{(192!)^{60}}\cdot 2^{11519}\right)^{\frac{\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{6}\left(2^{n \bmod 2}+2\right)}\left(\frac{2880!}{(48!)^{60}}\cdot 2^{2879}\right)^{(n \bmod 2)\left[\frac{(n-5)(n-3)}{8}\right]}\left(\frac{23040!}{(192!)^{120}}\right)^{\frac{\left\lfloor\frac{n-8}{2}\right\rfloor\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{24}}\cdot$$

$$\left(\frac{384!}{(32!)^{12}}\right)^{\left\lfloor\frac{n-2}{2}\right\rfloor}\left(\frac{960!}{(80!)^{12}}\right)^{(n \bmod 2)\left(n-3+\frac{(n-5)(n-3)}{8}\right)}\left(\frac{1920!}{(160!)^{12}}\right)^{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}\cdot$$

$$\left(\frac{3840!}{(320!)^{12}}\right)^{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor\cdot 2^{(n \bmod 2)-1}+\frac{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{2}+(n \bmod 2)\left[\frac{(n-5)(n-3)}{8}\right]}\left(\frac{480!}{(40!)^{12}}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\cdot$$

$$\left(\frac{5760!}{(480!)^{12}}\right)^{(n \bmod 2)\left[\frac{(n-5)(n-3)}{8}+\frac{(n-7)(n-5)(n-3)}{48}\right]}\left(\frac{11520!}{(960!)^{12}}\right)^{\frac{\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{6}\left(2^{(n \bmod 2)+1}+1\right)+(n \bmod 2)\left[\frac{(n-7)(n-5)(n-3)}{48}\right]}\cdot$$

$$\left(\frac{2880!}{(240!)^{12}}\right)^{(n \bmod 2)\left[\frac{(n-5)(n-3)}{4}\right]}\left(\frac{23040!}{(1920!)^{12}}\right)^{\frac{\left\lfloor\frac{n-8}{2}\right\rfloor\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{24}\left(2^{n \bmod 2}+3\right)+\frac{\left\lfloor\frac{n-10}{2}\right\rfloor\left\lfloor\frac{n-8}{2}\right\rfloor\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{60}}\cdot$$

$$\left(\frac{120!}{(10!)^{12}}\right)^{(n \bmod 2)\left(\frac{n-3}{2}\right)}\left(\frac{15360!}{(1280!)^{12}}\right)^{\frac{\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{6}}\left(\frac{7680!}{(640!)^{12}}\right)^{\frac{\left\lfloor\frac{n-6}{2}\right\rfloor\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{3}}$$

$$\psi(\mathrm{a},\mathrm{b}) = 2^{\mathrm{a}}\binom{b}{a}$$

$$\mathrm{z}(x_1, x_2, x_3, x_4, x_5) = \frac{\left(x_1 \cdot \psi(d-a_1,d)\prod_{j=1}^{i-1}\psi(a_j - a_{j+1}, a_j)\right)! \left[\frac{(d-a_1)!}{x_3}\right]^{x_1 \cdot \psi(d-a_1,d)\prod_{m=1}^{i-1}\psi(a_m - a_{m+1}, a_m)}}{\left[\left(x_2 \cdot \prod_{k=1}^{i-1}\psi(a_k - a_{k+1}, a_k)\right)!\right]^{x_3 \cdot \psi(d-a_1,d)} \cdot 2^{x_4} \cdot 3^{x_5}}$$

$$y_1 = \left[\prod_{a_i=1}^{a_{i-1}-1}\left(\mathrm{z}\left(1,1,1,\left\lceil\frac{d-a_1-1}{d-1}\right\rceil,0\right)\right)^{(n \bmod 2)\binom{\frac{n-3}{2}}{i-1}}\right]$$

$$y_2 = \left[\prod_{a_i=0}^{0}\left(\mathrm{z}\left(1,1,1,\left\lceil\frac{d-a_1-1}{d-1}\right\rceil,0\right)\right)^{\binom{\left\lfloor\frac{n-2}{2}\right\rfloor}{i-1}}\right]$$

$$y_3 = \left[\prod_{a_i=1}^{1}\left(\mathrm{z}\left(2,2,1,\left\lceil\frac{d-a_1-1}{d-1}\right\rceil,0\right)\right)^{\binom{\left\lfloor\frac{n-2}{2}\right\rfloor}{i-1}}\right]$$

$$y_4 = \left\{\prod_{a_i=2}^{2}\left(\mathrm{z}\left(4,4,1,\left\lceil\frac{d-a_1-1}{d-1}\right\rceil,0\right)\right)^{\binom{\left\lfloor\frac{n-2}{2}\right\rfloor}{i}\cdot 2^{n \bmod 2}}\left\{\left(\mathrm{z}\left(8,8,1,\left\lceil\frac{d-a_1-1}{d-1}\right\rceil,0\right)\right)^{\left\lceil\frac{a_1-i-1}{a_1-1}\right\rceil}\left[\left(\mathrm{z}\left(8,4,2,0,\left\lfloor\frac{4}{\mathrm{d}-\mathrm{a}_1}\right\rfloor\left\lceil\frac{d-a_1-2}{d-a_1}\right\rceil\right)\right)^{\left\lceil\frac{d-i-2}{d-2}\right\rceil}\left(\mathrm{z}(4,4,1,0,0)\right)^{2\left\lfloor\frac{i}{d-2}\right\rfloor}\right]^{\left\lfloor\frac{i}{a_1-1}\right\rfloor}\right\}^{\binom{\left\lfloor\frac{n-2}{2}\right\rfloor}{i+1}}\right\}$$

$$\mathrm{C_d}(n) = \frac{\left(2^d-1\right)!\left(\frac{d!}{2}\right)^{2^d-1}}{2^{\left\lceil\frac{d-3}{d}\right\rceil}\cdot 3^{\left\lfloor\frac{4}{d}\right\rfloor}}\left[2^{4-d-\left\lfloor\frac{3}{d}\right\rfloor}\prod_{i=2}^{d-1}\left(\psi(i,d)\right)!\left(i!\right)^{\psi(i,d)}\right]^{n \bmod 2}\left\{\frac{\left(2^d \cdot d\right)!\left[\frac{(d-1)!}{2}\right]^{2^d \cdot d}}{\left(2 \cdot 3^{\left\lfloor\frac{5}{d}\right\rfloor}\right)^{\left\lceil\frac{d-3}{d}\right\rceil}}\right\}^{\left\lfloor\frac{n-2}{2}\right\rfloor}\left\{\frac{\left[2^{d-1}\cdot d(d-1)\right]!\left[(d-2)!\right]^{2^{d-1}\cdot d(d-1)}}{2^{\left\lceil\frac{d-3}{d}\right\rceil}\cdot 24^{2^{d-3}\cdot d(d-1)}}\right\}^{\left\lfloor\frac{n-2}{2}\right\rfloor\cdot 2^{n \bmod 2}}.$$

$$\left\{\frac{\left[2^{d-\left\lfloor\frac{3}{d}\right\rfloor}\cdot d(d-1)\right]!\left\lceil\frac{(d-2)!}{2}\right\rceil^{2^{d-\left\lfloor\frac{3}{d}\right\rfloor}\cdot d(d-1)}}{3^{\left\lfloor\frac{6}{d}\right\rfloor\left\lceil\frac{d-4}{d}\right\rceil}\cdot 24^{2^{d-2-\left\lfloor\frac{3}{d}\right\rfloor}\cdot d(d-1)}}\right\}^{\frac{\left\lfloor\frac{n-4}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor}{2^{\left\lceil\frac{d-3}{d}\right\rceil}}} \qquad \prod_{i=2}^{d-2}\ \prod_{a_1=i+1}^{d-1}\prod_{a_2=i}^{a_1-1}\ \prod_{a_3=i-1}^{a_2-1}\cdots\prod_{a_{i-1}=3}^{a_{i-2}-1}\left(y_1 \cdot y_2 \cdot y_3 \cdot y_4\right)$$